Plaisirs (mathématiques) solitaires

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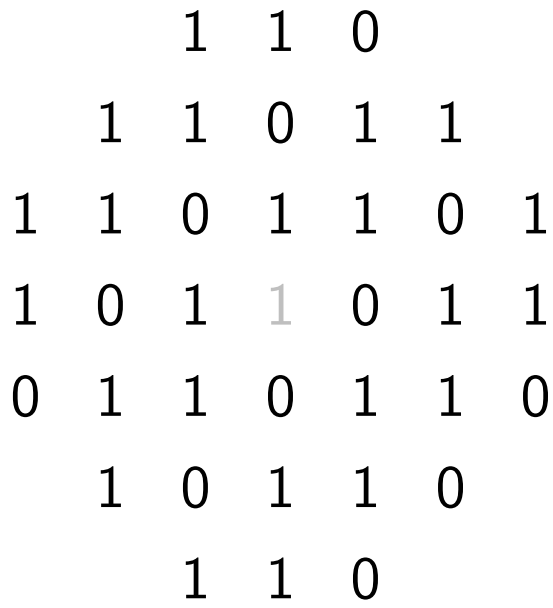
ISEN
ALL IS DIGITAL!
LILLE

yncréa

CLUBMATH
de l'Université de Montréal

11 octobre 2018

Princesse Soubise, 1697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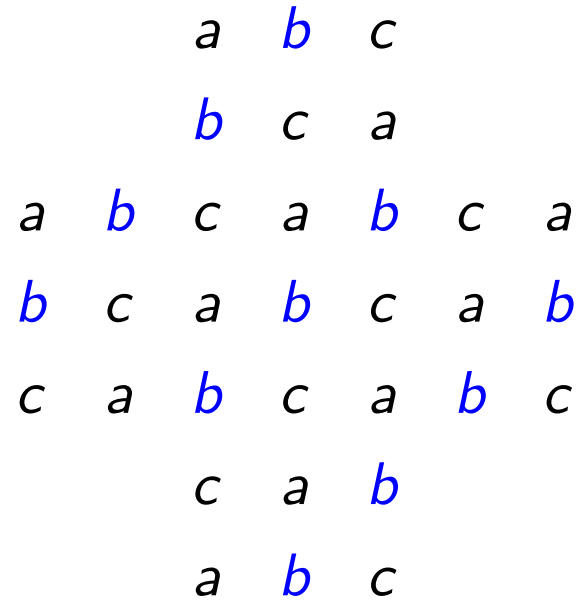
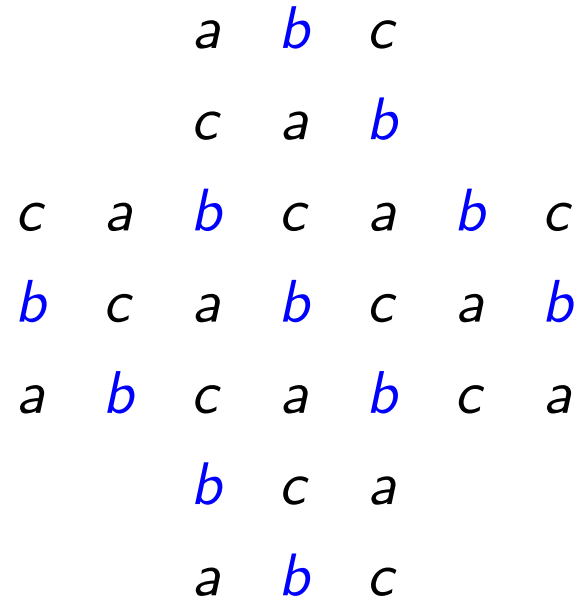
1 1 0 1 1 0 1
1 0 1 1 0 1 1
0 1 1 0 1 1 0
1 0 1 1 0
1 1 0

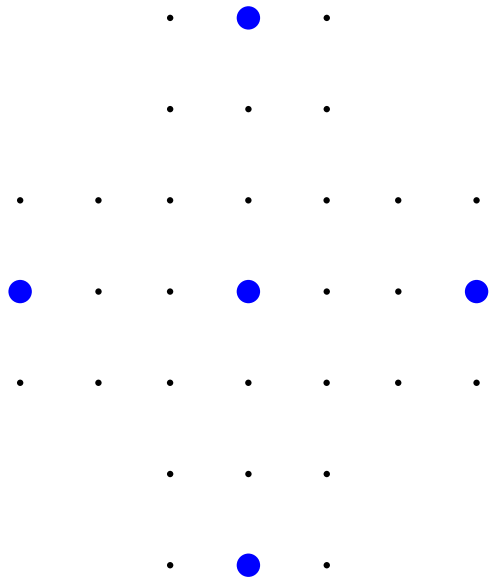
		a	b	c		
	a	b	c	a	b	
a	b	c	a	b	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
	b	c	a	b	c	
		a	b	c		

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$$
\begin{array}{ccccccc}
 & & a & b & c & & \\
 & & b & c & a & & \\
a & b & c & a & b & c & a \\
b & c & a & b & c & a & b \\
c & a & b & c & a & b & c \\
 & & c & a & b & & \\
 & & a & b & c & &
\end{array}
$$


a b c
b c a
a b c a b c a
b c a b c a b
c a b c a b c
c a b
a b c


a b c
c a b
c a b c a b c
b c a b c a b
a b c a b c a
b c a
a b c



$$
\begin{array}{ccccccc}
\alpha^0 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\
\alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \\
\alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 \\
\alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 \\
\alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 & \alpha^{10} \\
\alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 & \alpha^{10} & \alpha^{11} \\
\alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 & \alpha^{10} & \alpha^{11} & \alpha^{12}
\end{array}
$$

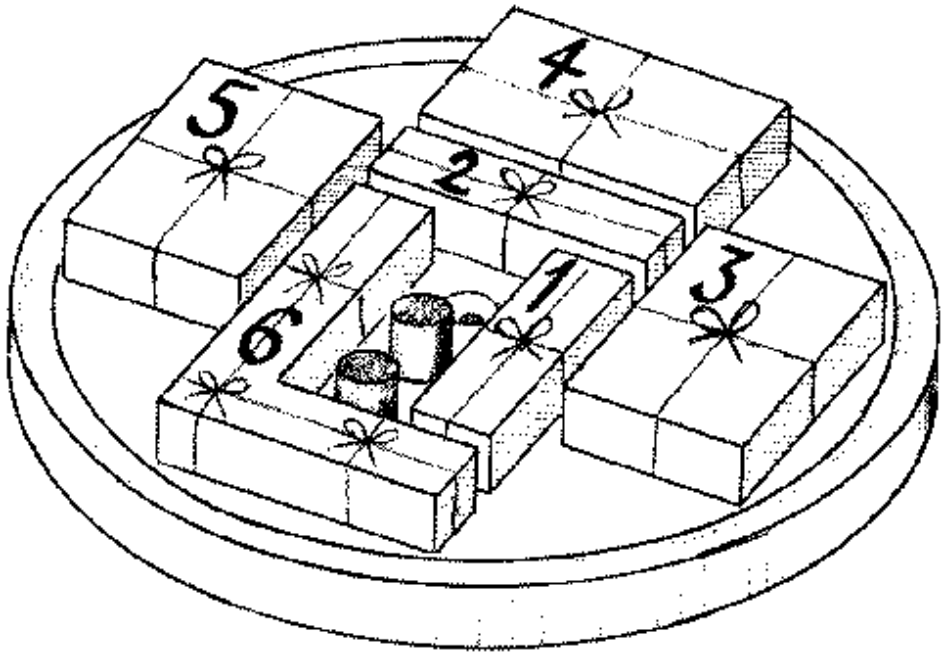
$+$	0	1	α	α^2
0	0	1	α	α^2
1	1	0	α^2	α
α	α	α^2	0	1
α^2	α^2	α	1	0

$\cdot$	0	1	α	α^2
0	0	0	0	0
1	0	1	α	α^2
α	0	α	α^2	1
α^2	0	α^2	1	α

$$
\begin{array}{ccccccc}
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 \\
\alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 & \alpha \\
\alpha^2 & 1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 \\
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 \\
\alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 & \alpha \\
\alpha^2 & 1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 \\
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1
\end{array}
$$

1	α	α^2	α^3	α^4	α^5	α^6
β	$\alpha\beta$	$\alpha^2\beta$	$\alpha^3\beta$	$\alpha^4\beta$	$\alpha^5\beta$	$\alpha^6\beta$
β^2	$\alpha\beta^2$	$\alpha^2\beta^2$	$\alpha^3\beta^2$	$\alpha^4\beta^2$	$\alpha^5\beta^2$	$\alpha^6\beta^2$
β^3	$\alpha\beta^3$	$\alpha^2\beta^3$	$\alpha^3\beta^3$	$\alpha^4\beta^3$	$\alpha^5\beta^3$	$\alpha^6\beta^3$
β^4	$\alpha\beta^4$	$\alpha^2\beta^4$	$\alpha^3\beta^4$	$\alpha^4\beta^4$	$\alpha^5\beta^4$	$\alpha^6\beta^4$
β^5	$\alpha\beta^5$	$\alpha^2\beta^5$	$\alpha^3\beta^5$	$\alpha^4\beta^5$	$\alpha^5\beta^5$	$\alpha^6\beta^5$
β^6	$\alpha\beta^6$	$\alpha^2\beta^6$	$\alpha^3\beta^6$	$\alpha^4\beta^6$	$\alpha^5\beta^6$	$\alpha^6\beta^6$

$$
\begin{array}{ccccccc}
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 \\
\beta & \alpha\beta & \alpha^2\beta & \beta & \alpha\beta & \alpha^2\beta & \beta \\
\beta^2 & \alpha\beta^2 & \alpha^2\beta^2 & \beta^2 & \alpha\beta^2 & \alpha^2\beta^2 & \beta^2 \\
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1 \\
\beta & \alpha\beta & \alpha^2\beta & \beta & \alpha\beta & \alpha^2\beta & \beta \\
\beta^2 & \alpha\beta^2 & \alpha^2\beta^2 & \beta^2 & \alpha\beta^2 & \alpha^2\beta^2 & \beta^2 \\
1 & \alpha & \alpha^2 & 1 & \alpha & \alpha^2 & 1
\end{array}
$$


5
4
2
1
3
6

		1	1	1		
		1	2	1		
1	1	2	3	2	1	1
1	2	3	5	3	2	1
1	1	2	3	2	1	1
		1	2	1		
		1	1	1		

$$
\begin{array}{ccccccccc}
 & & 3 & 2 & 1 & 0 & 1 & 2 & 3 & \\
 & & 4 & 3 & 2 & 1 & 2 & 3 & 4 & \\
\ldots & & 5 & 4 & 3 & 2 & 3 & 4 & 5 & \ldots \\
 & & 6 & 5 & 4 & 3 & 4 & 5 & 6 & \\
 & & 7 & 6 & 5 & 4 & 5 & 6 & 7 & \\
 & & & & & \vdots & & & &
\end{array}
$$

Références

- John Beasley, *The Ins & Outs of Peg Solitaire*
- *George's Peg Solitaire Page*
- Jaap Scherphuis, *Analysis of Peg Solitaire*
- Conway, Berlekamp & Guy, *Winning Ways for Your Mathematical Plays*, vol. 4